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Nonlinear acoustoelectric effect in a semiconductor superlattice

S Y Mensah†‡¶, F K A Allotey‡§ and N G Mensah‖

† Department of Physics, Laser and Fibre Optics Centre, University of Cape Coast, Cape Coast, Ghana
‡ The Abdus Salam International Centre for Theoretical Physics, Trieste, Italy
§ National Centre for Mathematical Sciences, Ghana Atomic Energy Commission, Kwabenya, Accra, Ghana
‖ Department of Mathematics, University of Cape Coast, Cape Coast, Ghana

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Abstract. The acoustoelectric effect in a semiconductor superlattice (SL) has been studied for a hypersound in the region $ql \gg 1$ (where q is the acoustic wave number and l is the electron mean free path). A nonlinear dependence of the acoustoelectric current j^{ac} on the constant electric field E is observed. It is noted that when the electric field is negative the current j^{ac} rises, reaches a peak and falls off. On the other hand, when the electric field is positive the current decreases, reaches a minimum and then rises. A similar observation has been noted for an acoustoelectric interaction in a multilayered structure resulting from the analysis of Si/SiO_2 structure. The dominant mechanism for such a behaviour is attributed to the periodicity of the energy spectrum along the SL axis.

1. Introduction

Electron transport in a semiconductor superlattice (SL) or multiquantum well structure is usually governed by drift, diffusive and tunnelling current flow. A ballistic carrier motion can also be observed, provided that the size of the system is smaller than the mean free path, ranging up to some hundred microns in some pure material. Conceptually different from these mechanisms are transport phenomena based on energy and momentum transfer from externally propagating entities to the electron medium. Such dragging experiments have been studied in great detail in double electron layer systems [1] where internal 'Coulomb friction' between the two layers causes the dragging force. Photon drag induced transport was observed for intersubband transitions of a quasi-two-dimensional electron system (2DES) [2]. In [3], Nagamune *et al* observed the effect of a dc current on the drift of optically generated carriers in a quantum well. Another interesting mechanism based on the transfer of energy and momentum is the interaction of acoustic phonons with carrier charges in semiconductor materials. This mechanism occurs not only during the scattering of quasi-momentum carriers by lattice vibrations but also when acoustic waves propagate through the material. Among the effects observed are absorption (amplification) of acoustic waves [4–7], the acoustoelectric effect (AE) [8–13], the acoustomagnetoelectric effect (AME) [14–18], the acoustothermal effect [19] and the acoustomagnetothermal effect [19].

¶ Regular Associate of the Abdus Salam ICTP.

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These phenomena have, however, received very little attention in SLs even though they have immense device applications. The interaction between a surface acoustic wave (SAW) and mobile charges in semiconductor layered structures has become an important method to study the dynamic conductivity of low-dimensional systems in quantum wells. These studies include the quantum Hall effect [20], the fractional quantum Hall effect [21], Fermi surfaces of composite Fermion around a half-filled Landau level [22] and commensurability effects caused by the lateral superlattice induced by a SAW [23]. It has also been noted that the transverse acoustoelectric voltage (TAV) is sensitive to the mobility and to the carrier concentration in the semiconductor, thus it has been used to provide a characterization of electric properties of semiconductors [24]. Interface state density [25], junction depth [26] and carrier mobility [27] have been measured with this method. Recently TAV has been proposed as an effective tool in the measurement of microstructures and superlattice parameters [28–31].

In this paper we shall endeavour to extend [32] to cover the nonlinear relation of the acoustoelectric current with the constant electric field. The acoustoelectric effect is the transfer of momentum from the acoustic waves to the conduction electrons as a result of which may give rise to a current usually called the acoustoelectric current j^{ac} or in the case of an open circuit, a constant electric field E^{ac}.

The study of this effect is vital because of the complementary role it may play in the understanding of the properties of the SL which we believe should find an important place in the acoustoelectronic devices. Experimental evidence of the dependence of the acoustoelectric effect on the parameters of SL has been reported in [33]. In [31] experimental work on acoustoelectric interaction of an SAW in a GaAs–InGaAs SL has been reported. A theoretical model for measuring the TAV in a multilayered structure resulting from the analysis of the Si/SiO_2 has also been reported in [34]. In that work analytical results were compared with experiments.

In this work, it will be shown that the presence of minibands in the SL will result in a nonlinear dependence of the j^{ac} and the ratio j^{ac}/Γ (where Γ is the absorption coefficient) on the wave number q. Also, in the presence of an applied constant electric field E a threshold value E_0 is obtained where the acoustoelectric current changes sign. Finally it will be noted that the acoustoelectric current rises and reaches a peak then drops in a manner similar to the negative differential conductivity observed in SL in the presence of constant electric field.

This paper is organized as follows. In section 2 we outline the theory and conditions necessary to solve the problem. In section 3 we discuss the results and finally in section 4 we draw some conclusions.

2. Theory

Proceeding as in [32], we calculate the acoustoelectric current in SL. The acoustic wave will be considered as a hypersound in the region $ql \gg 1$ (l is the electron mean free path, q is the acoustic wave number) and then treated as a packet of coherent phonons (monochromatic phonons having a δ-*function* distribution

$$N(k) = \frac{(2\pi)^3}{\hbar\omega_q s}\phi\delta(k-q) \qquad \hbar = 1 \tag{1}$$

where k is the phonon wavevector, $\hbar$ is Planck's constant divided by 2π, ϕ is the sound flux density and ω_q and s are respectively the frequency and the group velocity of sound wave, with the wavevector q.

It is assumed that the sound wave and the applied constant electric field E propagate along the z axis of the SL. The problem will be solved in the quasi-classical case, i.e. $2\Delta \gg \tau^{-1}$, $eEd \ll 2\Delta$ (τ is the relaxation time, d is the period of the SL, 2Δ is the width of the lowest energy miniband and e is the electron charge). The density of the acoustoelectric current can then be written in the form [17]

$$j^{ac} = \frac{2e}{(2\pi)^3}\int U^{ac}\Psi_i(p)\,\mathrm{d}^3p. \tag{2}$$

Here $\Psi_i(p)$ is the solution of the Boltzmann kinetic equation in the absence of a magnetic field; p is the electron momentum and

$$U^{ac} = -\frac{2\pi\phi}{\omega_q s}|G_{p-q,p}|^2[f(\epsilon_{p-q}) - f(\epsilon_p)]\delta(\epsilon_{p-q} - \epsilon_p + \omega_q) \\ +|G_{p+q,p}|^2[f(\epsilon_{p+q}) - f(\epsilon_p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q) \tag{3}$$

where $G(p,q)$ is the matrix element of the electron–phonon interaction and $f(\epsilon_p)$ is the distribution function. If we introduce a new term $p' = p - q$ in the first term of the integrals in equation (3) and take account of the fact that

$$|G_{p',p}|^2 = |G_{p,p'}|^2 \tag{4}$$

we can express equation (2) in the form

$$j^{ac} = -\frac{e\phi}{2\pi^2 s\omega_p}\int |G(p,q)|^2[f(\epsilon_{p+q}) - f(\epsilon_p)][\Psi_i(p+q) - \Psi_i(p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q)\,\mathrm{d}^3p \tag{5}$$

where the vector $\Psi_i(p)$, as indicated in [35], is the mean free path $l_i(p)$.

Thus the acoustoelectric current in equation (5) in the direction of the SL axis becomes

$$j_z^{ac} = -\frac{e\phi}{2\pi^2 s\omega_p}\int |G(p,q)|^2[f(\epsilon_{p+q}) - f(\epsilon_p)[l_z(p+q) - l_z(p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q)\,\mathrm{d}^3p. \tag{6}$$

For $qd \ll 1$, $G(p_z, q)$ is given as

$$|G(p_z,q)|^2 = \frac{\Lambda^2 q^2}{2\sigma\omega_q} \tag{7}$$

Λ is the deformation potential constant, and σ is the density of the SL. As indicated in [32], in the τ approximation and further, when τ is taken to be constant, l_z is given as

$$l_z = \tau v_z \tag{8}$$

where

$$v_z = \frac{\partial\varepsilon}{\partial p_z}. \tag{9}$$

The most convincing argument in favour of this condition is in [37], where it is established experimentally that the relaxation time τ is a constant in a GaAs/AlAs SL above 40 K and is temperature independent.

Inserting equations (7) and (8) into equation (6), we obtain

$$j_z^{ac} = -\frac{e\phi|\Lambda|^2 q^2\tau}{4\pi^2 s\omega_q^2}\int [f(\epsilon_{p+q}) - f(\epsilon_p)][v_z(p+q) - v_z(p)]\delta(\epsilon_{p+q} - \epsilon_p - \omega_q)\,\mathrm{d}^3p. \tag{10}$$

The distribution function in the presence of the applied constant field E is obtained by solving the Boltzmann equation in the τ approximation. This is given by

$$f(p) = \int_0^\infty \frac{\mathrm{d}t}{\tau} \exp(-t/\tau) f_0(p - eEt). \tag{11}$$

Here

$$f_0(p) = \frac{\pi dn}{mT I_0(\Delta/T)} \exp(-\epsilon_p/T) \tag{12}$$

where n is the electron density, T is the temperature in energy units and $I_0(x)$ is the modified Bessel function.

The key physical parameter describing the electron distribution in the bands is the dispersion relation, and for superlattices the dispersion law is given by

$$\epsilon_\nu(p) = \frac{p_\perp^2}{2m} + \epsilon_\nu - \Delta_\nu \cos(p_z d). \tag{13}$$

In equation (13), $p_\perp$ and p_z are the transverse and longitudinal (relative to the SL axis) components of the quasi-momentum, respectively; Δ_ν is the half width of the νth allowed miniband; ϵ_ν, given by

$$\epsilon_\nu = \frac{\hbar^2}{2m}\left(\frac{\pi}{d_0}\right)^2 \nu^2 \tag{14}$$

are the size-quantized levels in an isolated conduction film; $d = d_0 + d_1$ (d_0 is the width of the rectangular potential wells and d_1 is the barrier width with a non-zero quantum transparency) is the SL period.

We assume that electrons are confined to the lowest conduction miniband ($\nu = 1$) and omit the miniband indices. This is to say that the field does not induce transitions between the filled and empty minibands. The electron velocity is given by

$$v_z(p) = \frac{\partial\epsilon(p)}{\partial p_z} = \Delta d \sin(p_z d). \tag{15}$$

We further assume that $\epsilon_\nu = \Delta_\nu$ and write equation (13) in the usual form as

$$\epsilon(p) = \frac{p_\perp^2}{2m} + \Delta(1 - \cos(p_z d)). \tag{16}$$

Substituting equations (11), (15) and (16) into equation (5) and solving for a non-degenerate electron gas, we obtain for the acoustoelectric current

$$\begin{aligned} j_z^{ac} = {} & -\frac{e\phi|\Lambda|^2 q^2 n d\tau\theta(1-b^2)}{\sigma s\omega_q^2}\int_0^\infty \frac{\mathrm{d}t}{\tau}\exp(-t'/\tau) \\ & \times\left\{\sinh\left(\frac{\omega_q}{2T}\cos(eE\,\mathrm{d}t')\right)\sinh\left(\frac{\Delta}{T}\cos\left(\frac{qd}{2}\right)\cos(eE\,\mathrm{d}t')\sqrt{1-b^2}\right)\right. \\ & -\frac{\Delta}{T}\sqrt{1-b^2}\sin(eE\,\mathrm{d}t')\sin\left(\frac{qd}{2}\right)\cosh\left(\frac{\omega_q}{2T}\cos(eE\,\mathrm{d}t')\right) \\ & \left.\times\cosh\left(\frac{\Delta}{T}\sqrt{1-b^2}\cos\left(\frac{qd}{2}\right)\cos(eE\,\mathrm{d}t')\right)\right\} \end{aligned} \tag{17}$$

where θ is the Heaviside step function, and $b = \omega_q/[2\Delta\sin(qd/2)]$.

3. Results and discussion

We shall solve equation (17) for two particular cases.

(i) In the absence of the applied constant field ($E = 0$), from equation (17) we obtain

$$j_z^{ac} = \frac{e\phi|\Lambda|^2 q^2 n d\tau\theta(1-b^2)}{4\sigma s\omega_q^2}\sinh\left(\frac{\omega_q}{2T}\right)\sinh\left(\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\right). \quad (18)$$

It can be observed from equation (18) that, when $\omega_q \gg 2\Delta\sin(qd/2)$, $j_z^{ac} = 0$, i.e. there appears a transparency window. This is a consequence of the conservation law. Under this condition there is no absorption of acoustic waves hence no acoustoelectric current [36]. The SL can therefore be used as an acoustic wave filter. A similar suggestion has been made in [38] where the authors studied experimentally stimulated phonon emission in SL. It is also observed that the dependence of j_z^{ac} on q is strongly nonlinear. The ratio j_z/Γ_z, where Γ_z is calculated under the same conditions as in [36], is given by

$$\frac{j_z}{\Gamma_z} = -\frac{2e\phi\tau d\Delta\sin(qd/2)}{\omega_q}\sqrt{1-b^2}\tanh\left(\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\right). \quad (19)$$

Hence, we observe that j_z/Γ_z depends on the wave number q, i.e. it has a spatial dispersion. This behaviour is unlike the homogeneous semiconductor (bulk material) which is independent of q.

It is worth noting that, at $\Delta \gg T$, when the SL is behaving as a homogeneous semiconductor, j_z/Γ_z, as given in equation (19) satisfies the Weinreich relation

$$\frac{j_z}{\Gamma_z} = -\frac{e\phi\tau}{ms}. \quad (20)$$

As started in [35] the change in sign of the acoustoelectric current can be attributed to the fact that the main contribution to the acoustoelectric current at $qd \simeq 2\pi$ is from electrons near the top of the miniband, i.e. by electrons with a negative mass.

(ii) In a weak constant electric field, $eEd\tau \ll 1$, $\omega_q \ll T$, and from equation (17) we obtain

$$j_z^{ac} = j_0^{ac}\left\{1 - \frac{eEd\tau}{\omega_q}\sqrt{(2\sin(qd/2))^2 - \omega_q}\coth\left(\frac{\Delta}{T}\cos(qd/2)\sqrt{1-b^2}\right)\right\}. \quad (21)$$

From equation (21) it is observed that at

$$E > E_0 = \omega_q\frac{\tanh((\Delta/T)\cos(qd/2)\sqrt{1-b^2})}{e\tau d\sqrt{(2\sin(qd/2))^2 - \omega_q}} \quad (22)$$

the acoustoelectric current changes sign. The value E_0 can be interpreted as a threshold field. E_0 is a function of the SL parameters d and Δ, temperature T, frequency ω_q and the wavenumber q. For example, at $\Delta/T \ll 1$, $\Delta = 0.1$ eV, $d = 5\times10^{-7}$ cm, $\tau = 10^{-12}$ s, $s = 5\times10^5$ cm s^{-1} and $\omega_q = 10^{10}$ s^{-1}. For these values we obtain the threshold field $E_0 = 8.65$ V cm^{-1} which is small and can be observed.

The general solution of equation (17) cannot be solved analytically. We, therefore, obtained its solution numerically, and the graph of j_z^{ac} against E was plotted (see figures 1 and 2). The dependence of j_z^{ac} on E for given Δ is presented in figure 1. It is noted that the acoustoelectric current has a peak at some values of E. These peaks decrease with a corresponding decrease of Δ. More interesting is the nature of the acoustoelectric current.

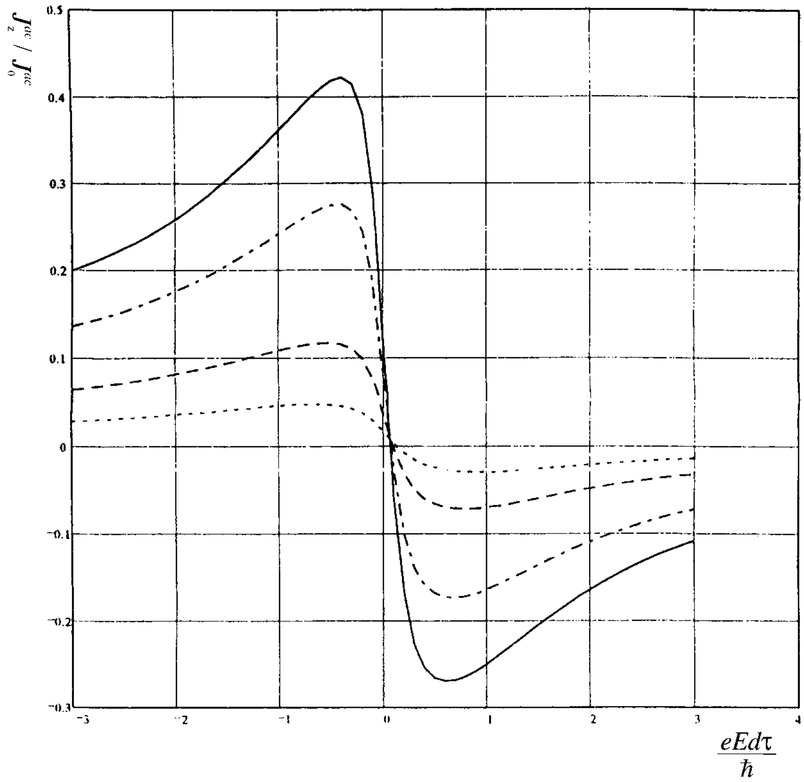

Figure 1. Dependence of J^{ac}/J^{ac}_0 on $eEd\tau/\hbar$: ——, $\Delta = 0.1$ eV; — · —, $\Delta = 0.09$ eV; − − −, $\Delta = 0.07$ eV; · · · · · ·, $\Delta = 0.05$ eV.

It is observed that when the electric field is negative the current rises, reaches a maximum and then falls off in a manner similar to that observed during a negative differential conductivity. On the other hand, when the electric field is positive the current decreases, reaches a minimum then increases. This can be attributed to the Bragg reflection at the band edge. It is further observed that the ratio of the height of the peak corresponding to absorption to that corresponding to amplification differ from one. This value also decreases with a decrease in Δ. The threshold field E_0 also increases with the decrease of Δ. It is noteworthy to show that a similar nonlinear relation was obtained for a TAV experiment on Si/SiO_2 and this result agrees quite well with our result [34].

In figure 2 the dependence of j^{ac}_z on E is plotted for given ω_q. We observed a decrease and shift of the peak values as ω_q decreases. The threshold field E_0 also increases with increase in ω_q.

4. Conclusion

We have studied the acoustoelectric effect in an SL in the presence of constant electric field and noted a strong nonlinear dependence of j^{ac}_z on E which strongly depends on Δ and ω_q. The dominant mechanism for such nonlinear behaviour is the periodicity of the energy spectrum along the SL axis. We observed that a transparency window is formed whenever $\omega_q \gg 2\Delta \sin(qd/2)$, i.e. $j^{ac}_z = 0$. We attributed the cause to the presence of the conservation

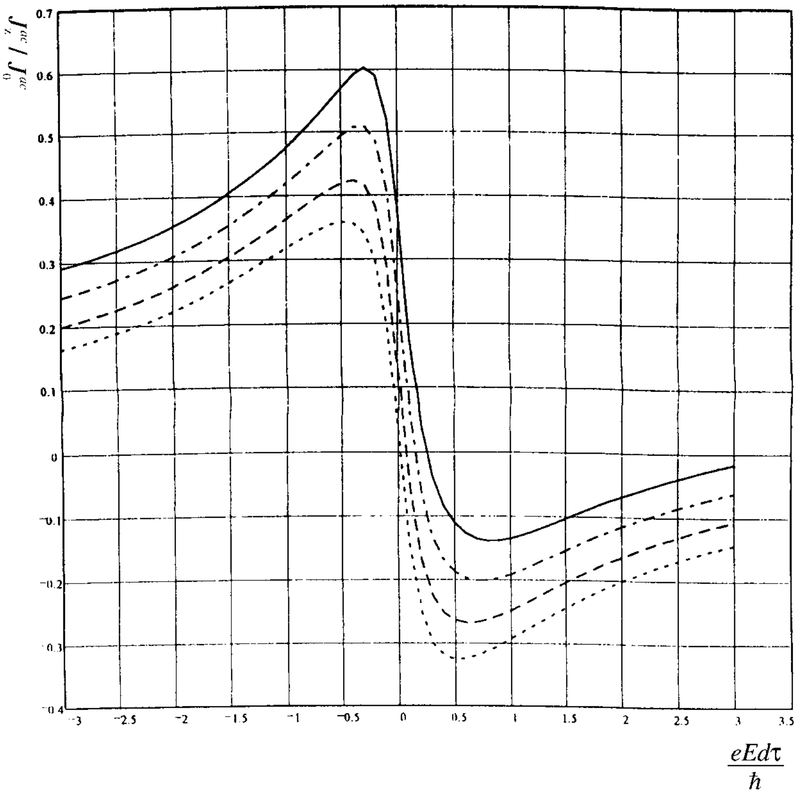

Figure 2. Dependence of J_z^{ac}/J_0^{ac} on $eEd\tau/\hbar$: ——, $\omega_q = 1.2 \times 10^{12}$ s^{-1}; — · —, $\omega_q = 7.8 \times 10^{11}$ s^{-1}; - - -, $\omega_q = 3.8 \times 10^{11}$ s^{-1}; · · · · · ·, $\omega_q = 7.9 \times 10^{11}$ s^{-1}.

laws and suggested the use of SL as a phonon filter. Finally, we noted that there exists a threshold field E_0 for which the acoustoelectric current changes sign and that this value increases with a decrease in Δ and increases with an increase in ω_q.

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